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Developmental Education
Content Committee:

Mathematics
Shared Learner Outcomes Report

April 2017

Minnesota State Colleges and Universities
Developmental Mathematics Shared Learner Outcomes:
Content, Processes and Practices that Develop Readiness
for College-Level Mathematics

Table of Contents

Introduction

This report was prepared in the 2016 – 2017 academic year by a committee of Minnesota State Colleges and Universities (MnSCU) mathematics educators. This group will be referred to as the Shared Learner Outcomes (SLO) Committee throughout the report. The SLO Committee consisted of Minnesota State College Faculty (MSCF) and Inter Faculty Organization (IFO) members. The charge of the SLO Committee was to:

1. Submit recommendations of specific competencies and student learner outcomes to be mastered for college and career readiness.
2. Make recommendations of how to promote seamless transition and transfer of developmental mathematics curriculum across the MnSCU system.
3. Make recommendations on how the MnSCU system can better work with Minnesota K-12 school districts on strategies of aligned competencies, student learner outcomes, and targeted interventions to promote college readiness.

The SLO Committee created this document describing shared learner outcomes in developmental mathematics with the intent to foster future conversations about shared learner outcomes for college level courses, transferability within the MnSCU system and collaborative work with Minnesota K-12 school districts.

History of the Work of the Shared Learner Outcomes Group

The SLO Committee began its work in the 2013-2014 academic year at the invitation of MnSCU's Office of the Vice-Chancellor of Academic Affairs. The focus of the work the initial year was on the development of shared understandings of Intermediate Algebra learner outcomes across the system. The SLO committee's work culminated in construction of a document presenting a description of the nature of the diverse contextual settings of developmental mathematics programs across the MnSCU system, a description of the intellectual processes and practices that are common in Intermediate Algebra, and a description of mathematics content outcomes for Intermediate Algebra. The 2013 – 2014 document serves as the basis for the SLO Committee's present work.

Developmental Mathematics Education across the MnSCU System

Developmental mathematics is pre-college level mathematics coursework offered by institutions of higher education. Developmental mathematics is central to providing access to higher education for students who need support in the development of essential content related competencies and practices for college level coursework and their overall higher education experience. The majority of developmental coursework that occurs in Minnesota public higher education institutions takes place within the MnSCU system, with the bulk of

developmental coursework occurring within two-year community or technical colleges (SLEDS, 2015).

Developmental mathematics programming within the MnSCU system is complex and varies from institution to institution. It is the belief of the SLO Committee that a strength of the MnSCU system is its ability to serve the needs of unique academic programs and diverse students in ways that support the quality of life and economic vitality of the state. The demands on an individual institution's developmental mathematics programs are a product of these diversities. Specific mathematical focuses that support an individual school's academic programs must be thoughtfully and skillfully embedded within developmental mathematics programs that effectively attend to the larger structures of mathematics with focus on understanding, connections, reasoning, transfer of learning and communication. The goals that attend to the unique needs of MnSCU programs and students are aligned with the principles of teaching, learning and curriculum outlined in the *Crossroads* (American Mathematical Association of Two-Year Colleges [AMATYC], 1995) and *Beyond Crossroads* (AMATYC, 2006) standards documents.

Local control of developmental mathematics curriculum across the MnSCU system is necessary to attend to the rigor and complexities of implementing strong curriculum in a diversity of contexts. Some of the parameters described in this document will be quite specific and may seem to be definitive of practice. This is seldom the case. Even where institutions align perfectly on some parameter, actual practice often varies considerably and the robust context of each environment must be considered.

"Developmental education is about treating students holistically and realizing that they're not just students taking courses but people who are parenting and working, many of whom are struggling to manage their finances and their time," says Hunter R. Boylan, director of the National Center for Developmental Education. (Mangan, 2015)

The diversity among developmental student populations illustrates the role of developmental programs in opening doors to higher education (Hardin, 1998). More students are attending college from a variety of backgrounds, and higher education is experiencing greater student diversity in terms of a wide range of student characteristics. In the mathematics area, those students benefiting from developmental courses include students:

- who are capable students who have simply fallen behind.
- who are generally prepared for college level study, but are underprepared in mathematics.
- who are motivated to pursue college level work, but may lack generalized learning skills as well as math-specific skills.
- who have verifiable (usually documented) learning disabilities.
- who have a limited proficiency in multiple areas including mathematical abilities, learning skills, motivation, organizational skills, and others. (NADE Math SPIN, 2002)

The mathematical content and practices that are taught and learned in developmental mathematics courses must be carefully considered and valued as the conversations surrounding developmental education progress. The stance of the SLO Committee is that Mathematics faculty within MnSCU have been, and will continue to be

leaders in reviewing the best practices, researching innovative directions and redesigning developmental mathematics programs to increase student success. Five important elements are involved in this effort: mathematical content and practices, course taking trajectories, pedagogy, student success factors and faculty development. The current document addresses the first of these, with recognition given to the equal importance of all five. Establishing content and practice standards for all developmental mathematics programs within MnSCU will provide a firm foundation for the ongoing development of curriculum, courses and programs in ways that support improved student outcomes.

Developmental Mathematics Courses

The developmental courses and course trajectories across the MnSCU system vary by institution. The differences observed across institutions are the result of the diversities of the programs and populations unique to each college or university. The present document outlines content and practice standards for developmental mathematics with the understanding that the particular course titles and content division may not fit the trajectories unique to individual institutions. The titles used in the present document to differentiate different course content standards are Prealgebra/Arithmetic, Elementary Algebra, and Intermediate Algebra. These titles were selected because they are a familiar trajectory often considered a traditional trajectory leading to college level mathematics.

When determining the "level" (Elementary Algebra versus Intermediate Algebra) of specific content outcomes, some variability can and will exist from campus to campus. It is understood that there is some overlap between courses at the level of Elementary and Intermediate Algebra. It may be helpful to consider the difference between Elementary Algebra and Intermediate Algebra more as a difference in the level of "mathematical maturity" rather than a difference in specific competencies. For example, some institutions may choose to cover systems of equations at the elementary level while others may choose to cover it at both the elementary and intermediate or entirely at the intermediate level, the difference being both the complexity of the procedural knowledge (computations requiring integers versus fractions, for example) and contextual knowledge (nature of applications and complexity of systems, two, three or more variables, or introducing matrix methods to computing solutions).

While a student may not have mastered or even been exposed to a specific competency, completion of a course at the "elementary algebra" level should give a student a sufficient mathematical background to facilitate comprehension of the mathematics typically encountered in a course at the level of "Intermediate Algebra". Similarly, completion of a course at the level of "Intermediate Algebra" will give a student a sufficient mathematical background to facilitate the comprehension of the mathematical concepts encountered in an entry level college mathematics course. While a student may possess a sufficient level of mathematical maturity to be successful in a course, many other factors contribute to the level of success students achieve in an entry level college mathematics course. The elements specified here address only the level of mathematical competencies and do not address these other factors.

Processes and Practices that Support Student Success in Mathematics

Developmental mathematics provides opportunities for students to establish important mathematical processes and practices that support and develop flexible and powerful understandings, proficiencies and expertise. The following list describes some of the particular mathematical processes and practices that are central to the learning experiences in developmental mathematics.

- Clear interpretation and representation of mathematical problems.
- Selection and construction of modes of representation appropriate to the understanding and reasoning processes within various mathematical problems.
- Flexible and applicable problem solving tools and practices.
- The development and value of rigor and detail in mathematical problem solving.
- Abstraction and generalization of mathematical problems to larger mathematical structures.
- Effective communication of mathematical ideas and understandings.
- Supportive learning environment for adult learners who need
    - to know what, why and how they learn
    - to use their previous experiences in learning new contents
    - to explore how subjects can be applied in their learning and life situation
    - to work on solving real-life problems

Mathematical learning experiences have a major role in the development of some broader higher-level skills and attributes that support student success in higher education. These skills and attributes are articulated in the following list.

- Improved problem solving tenacity.
- The ability to adopt multiple perspectives.
- Recognition of ambiguity and the development of the ability to seek deeper understanding and analysis.
- The use of context to inform an appropriate response in a given situations.
- The development of logical thought and process.
- The recognition and value of sound analysis.

The mathematical practices and processes outlined in the lists above champion the process, practice and development standards presented by The National Council of Teachers of Mathematics [NCTM] (2000), AMATYC (1995), and in The Common Core State Standards (Common Core State Standards Initiative [CCSSI], 2016).

Developmental Mathematics Content Standards and Shared Learner Outcomes

In this section of the report, the SLO Committee presents a list of content objectives that are characteristic of the curriculum found within developmental mathematics courses. The content objectives listed are consistent with the prerequisite mathematics content needed for success in college level math including College Algebra, Liberal Arts Mathematics and Statistics.

The content objective list was developed through consideration of state and national curriculum standards and principles presented by AMATYC (1995), NCTM (1989, 2000), NGSS (2016), and the Minnesota Department of Education [MDE] (2007). Additionally, a survey of learning objects found within developmental mathematics courses across the system (Appendix A) and documents from prior placement test cut score meetings (Appendix B) were also considered.

The following list does not define nor identify an intended curriculum. Such a definition of curriculum limits the learning of mathematics to rote and disconnected procedures and would bring the context of teaching and learning back many decades through another push to go "back to the basics." A quality curriculum moves beyond a narrowly defined set of skills. It attends to both the processes and products of doing mathematics. Curriculum cannot, and should not, be considered independent from the contexts of its implementation. The SLO Committee, therefore, offers the following list for consideration but makes no concerted effort to delineate each institution's approach to these matters. Appendix C offers examples and explanations of underlying strands of the objectives.

The following five content standards describe the content that developmental mathematics students should learn. The shared learner content outcomes that follow are organized by these standards. The SLO Committee believes that many of the outcomes fall within the intersection of the standards and that a strong curriculum centers on the connections of the standards through the teaching and learning of the outcomes.

A. Number and Operations

   Students will have a robust understanding of real numbers and demonstrate computational fluency with real and complex numbers. Strong number sense and meaningful understanding of arithmetic operations support the ability to estimate, reason and operate flexibly in quantitative situations and serve as the foundation of algebraic thinking.

B. Modeling

   Students will engage in modeling activities that are connected to contexts found in applied situations and settings. Modeling experiences support the development of mathematical reasoning and conceptual understanding and provide the opportunity to formulate, solve and reflect on mathematical problems and processes.

C. Symbolism and Algebraic Expressions

   Students will represent mathematical situations and structure symbolically, and flexibly operate with algebraic expressions. Connecting concepts and techniques to overarching algebraic structure through abstraction and generalization of patterns and relations supports meaningful understanding of mathematics as an integrated whole. Symbolic representation of mathematical concepts and structures and their

connections to graphical, numerical and contextual representations serve as an important component to the foundation of algebraic thinking and reasoning.

D. Algebraic Equations and Inequalities

Students will operate and reason within mathematical situations and contexts that are represented through algebraic equations and inequalities. Meaningful understanding of mathematical relationships that are defined through equality and inequality support the ability to flexibly approach the manipulation of such relationships in the processes of problem solving and modeling.

E. Functions and Relations

Students will demonstrate an understanding of the concepts of relations and functions through translations within and among multiple modes of representations including symbolic, graphical, numerical and contextual. Relations and functions are central structures in algebra, and understanding of their properties and generalizations about families of relations and functions support the development of mathematical thinking and problem solving.

These standards apply across all courses, prealgebra through intermediate algebra, though the emphasis of each standard differs by course.

Developmental Mathematics Outcomes for the College Algebra Path

The developmental mathematics outcomes needed prior to entering College Algebra are broken down into the three familiar course title of Prealgebra/Arithmetic, Elementary Algebra and Intermediate Algebra. It is understood that not all institutions utilize this separation of courses but rather an overall developmental mathematics sequence that encompasses all of these outcomes for a sequence commonly utilized to describe preparation for college algebra.

Prealgebra / Arithmetic Shared Learner Outcomes

A. Numbers and Operations

a. Classify real numbers as natural, whole, integer, rational and irrational.
b. Understand and interpret integers and decimals through the place value-structure of the base-10 number system. Round real numbers and use meaningful approximations of numbers to estimate values.
c. Compare and order real numbers. Relate real number values using inequality and equality relations.
d. Determine and interpret the absolute value of a real number.

e. Develop fluency in adding, subtracting, multiplying and dividing fractions and decimals.
f. Develop fluency with the order of operations applied to real number expressions including addition, subtraction, multiplication, division, grouping symbols, absolute value, integer exponents and roots.
g. Identify and apply the associative, commutative and distributive laws when operating with real number expressions.
h. Use factors, prime factorizations and multiples to solve problems. Determine the greatest common factor and least common multiple for sets of integers.

B. Modeling

a. Describe the shape and characteristics of data including measures of center (mean, median, mode) and range.
b. Recognize and apply geometric relationships such as perimeter, area and volume to solve problems.
c. Recognize right triangles and apply the Pythagorean Theorem.
d. Evaluate formulas for specified values and interpret the results in the context of an application.
e. Form and interpret ratio, and rate relationships quantitatively and contextually.
f. Utilize proportional relationships and percentages in contextualized applications.

C. Symbolism and Algebraic Expressions

a. Evaluate algebraic expressions involving addition, subtraction, multiplication, division, powers and square roots.

D. Algebraic Equations and Inequalities

a. Represent and operate with proportional contexts using proportions and the y = mx model.

Elementary Algebra Shared Learner Outcomes

A. Numbers and Operations

a. Operate flexibly with real number expressions using the order of operations including addition, subtraction, multiplication, division, grouping symbols, absolute value, integer exponents and roots.
b. Understand and use properties of real numbers including the associative, commutative and distributive laws, and inverse relationships when operating with real number expressions.

c. Simplify algebraic expressions.
d. Evaluate algebraic expressions.
e. Represent numbers using scientific notation and operate with numbers in scientific notation.

B. Modeling

a. Recognize and apply geometric relationships such as perimeter, area and volume to solve problems.
b. Understand and apply Pythagorean relationships to flexibly solve applied problems.
c. Recognize and apply angle relationships within geometric figures and intersecting lines.
d. Solve applied linear problems involving percent, uniform motion and mixture.
e. Explore the use of systems of linear equations in two variables to solve problems when applicable.
f. Recognize quadratic relationships and apply quadratic models to solve problems.
g. Solve applied problems involving inequality relationships.
Recognize linear relationships and apply linear models to solve problems using tables, graphs, words and equations.

C. Symbolism and Algebraic Expressions

a. Use rules of exponents to simplify expressions including integer exponents.
b. Develop fluency with adding, subtracting and multiplying polynomial expressions, including polynomials in several variables.
c. Divide polynomials by a monomial divisor.
d. Identify common factors within terms of polynomial and factor polynomials using greatest common factors and the method of factoring by grouping.
e. Factor trinomials in quadratic form.

D. Algebraic Equations and Inequalities

a. Solve linear inequalities and compound inequalities.
b. Solve linear equations in one variable.
c. Develop fluency in solving systems of linear equations using the methods of graphing, elimination and substitution.

E. Functions and Relations

a. Understand the structure of the Cartesian coordinate plane.
b. Represent linear relationships in two variables using graphs and use graphs to analyze and interpret constant rates of change.

c. Translate among multiple representations of patterns including contexts, tables, graphs and equations.

Intermediate Algebra Shared Learner Outcomes

A. Numbers and Operations

a. Represent, compare and classify real and complex numbers.
b. Operate flexibly with real number expressions using the order of operations including addition, subtraction, multiplication, division, grouping symbols, absolute value, rational exponents and roots.
c. Perform operations on complex numbers including addition, subtraction and multiplication.

B. Modeling

a. Utilize multiple representations including tables, graphs, words and symbols to explore and solve a variety of problems.
b. Write algebraic expressions, equations and inequalities to represent contextualized applications.
c. Solve applied mathematical problems involving polynomial models.
d. Solve applied mathematical problems involving rational and radical models.
e. Interpret solutions of problems in context and determine reasonableness of quantitative results.
f. Solve literal equations and interpret the contexts of formulas.

C. Symbolism and Algebraic Expressions

a. Use rules of exponents to simplify expressions including rational exponents.
b. Simplify algebraic radical expressions and perform operations with radicals.
c. Convert expressions between radical form and exponential form.
d. Add, subtract, multiply and divide polynomial expressions.
e. Factor polynomials using a variety of methods including grouping, special factoring (e.g. difference of squares, perfect square trinomials, sum and difference of cubes) and trial and error.
f. Add, subtract, multiply, divide and simplify rational expressions.

D. Algebraic Equations and Inequalities

a. Solve multistep linear equations and inequalities with rational coefficients.
b. Solve 2 x 2 systems of linear equations with rational coefficients using the methods of graphing, elimination and substitution.
c. Solve absolute value equations.

d. Solve rational equations.
e. Solve radical equations.
f. Solve quadratic equations with real and complex solutions using the following methods: factoring, the square root property, completing the square and the quadratic formula.

E. Functions and Relations

a. Translate between graphical, tabular, verbal and symbolic representations of functions and relations.
b. Identify the domain and range of functions and relations.
c. Recognize function notation and evaluate functions for specified domain values.
d. Evaluate a function for a specified value.
e. Perform algebraic operations with functions.
f. Recognize and graph linear functions.
g. Recognize and graph quadratic, absolute value, square root and exponential functions.

Preparation for Liberal Arts Mathematics

MnSCU schools offer a number of college level gateway courses that broadly could be titled Liberal Arts Mathematics. The content and consequently the pre requisite material for these courses vary widely as the courses support diverse academic programs unique to their institutions. Statistics is another college level gateway course offered by MnSCU institutions. Content and contextual variations in Statistics courses also vary by institutions according to the structures of their academic programs. One characteristic that Liberal Arts Mathematics and Statistics courses share is that students can be successful with fewer algebra skills than what is typically encountered in Intermediate Algebra.

MnSCU institutions have taken a variety of approaches to the developmental sequences supporting college mathematics coursework. For example, some institutions allow entry into Liberal Arts Mathematics with either the traditional Elementary Algebra course or they have shifted some algebra content from Intermediate Algebra to Elementary Algebra. Others require the same sequence as entry into College Algebra. Other institutions are able to create and run a separate trajectory that addresses some of the non – algebra topics that may lead to increased student success in a Liberal Arts Math or a Statistics course. While it may appear that the last approach would be preferred, there are trade-offs in doing so. Some schools simply cannot create a new sequence because of small enrollments. Another issue encountered when creating a separate course is that typically this new course does not meet the level of preparation necessary for entry into College Algebra. Students who might take this new course and then change their educational goals later may end up taking more developmental math but not gain any benefit from doing so.

The shared learner outcomes for the developmental mathematics for preparation for a college level Liberal Art Mathematics course.

Developmental Mathematics Outcomes for Liberal Arts Mathematics Path

A. Numbers and Operations

a. Classify real numbers as natural, whole, integer, rational and irrational.
b. Understand and interpret integers and decimals through the place value-structure of the base-10 number system. Round real numbers and use meaningful approximations of numbers to estimate values.
c. Compare and order real numbers. Relate real number values using inequality and equality relations.
d. Determine and interpret the absolute value of a real number.
e. Develop fluency in adding, subtracting, multiplying and dividing fractions and decimals.
f. Develop fluency with the order of operations applied to real number expressions including addition, subtraction, multiplication, division, grouping symbols, absolute value, integer exponents and roots.
g. Identify and apply the associative, commutative and distributive laws when operating with real number expressions.
h. Use factors, prime factorizations and multiples to solve problems. Determine the greatest common factor and least common multiple for sets of integers.
i. Operate flexibly with real number expressions using the order of operations including addition, subtraction, multiplication, division, grouping symbols, absolute value, rational exponents and roots.
j. Understand and use properties of real numbers including the associative, commutative and distributive laws, and inverse relationships when operating with real number expressions.
k. Simplify algebraic expressions.
l. Evaluate algebraic expressions.

B. Modeling

a. Describe the shape and characteristics of data including measures of center (mean, median, mode) and range.
b. Recognize and apply geometric relationships such as perimeter, area and volume to solve problems.
c. Recognize right triangles and apply the Pythagorean Theorem.
d. Evaluate formulas for specified values and interpret the results in the context of an application.
e. Form and interpret ratio and rate relationships quantitatively and contextually.

f. Utilize multiple representations including tables, graphs, words and symbols to explore and solve a variety of problems.
g. Utilize proportional relationships and percentages in contextualized applications.
h. Recognize linear relationships and apply linear models to solve problems using tables, graphs, words and equations.
i. Explore the use of systems of equations in two variables to solve problems when applicable.
j. Interpret solutions of problems in context and determine reasonableness of quantitative results.
k. Solve literal equations and interpret the contexts of formulas.
l. Use the basic operations with denominate numbers and unit conversions.

C. Symbolism and Algebraic Expressions

a. Evaluate algebraic expressions involving addition, subtraction, multiplication, division, powers and square roots.
b. Use rules of exponents to simplify expressions with integer exponents.
c. Divide polynomials by a monomial divisor.
d. Develop an initial understanding of adding, subtracting and multiplying polynomials.
e. Identify and factor the greatest common monomial within terms of a polynomial.

D. Algebraic Equations
a. Represent and operate with proportional contexts using proportions and the y = mx model.
b. Solve linear equations in one variable.

E. Functions and Relations
a. Understand the structure of the Cartesian coordinate plane.
b. Represent linear relationships in two variables using graphs and use graphs to analyze and interpret constant rates of change.
c. Translate among multiple representations of patterns including verbal descriptions, tables, graphs and equations.

Statistics is another college level gateway course offered by MnSCU institutions. Content and contextual variations in Statistics courses vary by institutions according to the structures and requirements of their academic programs. The discussion and organization of selecting outcomes for a developmental mathematics path to Statistics has begun, but additional work is needed to develop a document to be reviewed for consensus.

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Appendix A

Learner Outcomes -Math
Apply the Binomial Theorem
communicate clearly a problems solution and its explanation for the intended audience in terms of the problem posed.
Demonstrate appropriate manipulation of rational, radical, exponential and logarithmic expressions, as well as expressions invoving rational exponents
Demonstrate appropriate use of function notation and be able to find the domain and range of functions; as well as find the inverse of one-to-one functions
Demonstrate critical and logical reasoning when solving problems.
Demonstrate proficiency in basic arithmetic calculations and algebraic derivations, at the prerequisite level of Colle
Determine the sum, difference, product, quotient, composition, and inverse of functions.
doing complex number arithmetic
Evaluate expressions involving function notation
Evaluate functions.
factor polynomials
Factor sum and difference of cubes
Given a simple one-to-one function, find its corresponding inverse function
Graph a linear inequality in two variables
Graph a quadratic function, identifying the line of symmetry, vertex, and x-intercepts
Graph basic linear, quadratic, exponential, and logarithmic functions and transformations of these.
Model applications based on these functions
graph linear and non-linear inequalities and quadratic equations
Graph linear functions and vertical lines
Graph linear, quadratic, exponential, logarithmic functions by hand and with technology.
Graph parabolas and circles by hand.
Graph simple cubic and square root functions
Graph simple exponential functions
Graph simple logarithmic functions
Identify a function and its domain and range
Identify the domain and range of functions symbolically and graphically.
Illustrate problems.
manipulating expressions with rational exponents
manipulating logarithmic and exponential expressions
Model and solve applied problems using linear, rational, radical, quadratic, exponential, and logarithmic equations.
Model applications using the types of equations and inequalitites liest in 1
Perform basic operations on polynomials and rational expressions expressing the answer in simplified form. (omit
Perform basic operations on real numbers and addition, subtraction and multiplication on complex numbers
perform operations with functions
perform operations with radical expressions and rational expressions
perform operations with rational exponents
Represent applications using systems of linear equations
Simplify complex rational expressions
Simplify expressions containing radicals or rational exponents
Simplify polynomial, rational, radical, complex, exponential, and logarithmic expressions symbolically.
Simplify polynomials using properties of exponents and factoring

Simplify radicals and solve radical equations
Simplify rational expressions and solve rational equations
simplifying complex fractions
simplifying rational expressions and doing rational expression arithmetic
Solve a system of linear equations in two variables by substitution, elimination and graphing
Solve absolute value equations and inequalities
Solve algebraic equations and inequalities
solve equations containing radicals and rational expressions
Solve first degree equations and inequalities
Solve linear and absolute value equations
Solve linear compound, polynomial, rational, and absolute value inequalities symbolically.
Solve linear, quadratic, polynomial, rational, radical, exponential and logarithmic equations; as well as linear, polynomial, and rational inequalities
Solve polynomial, rational, absolute value, radical, exponential, and logarithmic equations symbolically.
Solve quadratic and absolute value inequalities
Solve quadratic equations by factoring, completing the square, and the quadratic formula
Solve quadratic equations by factoring, taking square roots, completing the square, and using the quadratic formu
solve quadratic equations by factoring, taking square roots, completing the square, and using the quadratic formula
Solve quadratic equations that have complex roots
Solve quadratic equations using factoring, the quadratic formula, square root method, or completing the square
Solve radical equations
Solve rational equations
Solve simple exponential equations
Solve simple logarithmic equations
Solve systems of linear equations
Solve systems of linear equations symbolically and graphically.
Solve systems of linear equations using matrices and model applications using systems of linear equations
Solve systems of linear inequalities graphically.
solving compound, absolute value, and quadratic inequalities
solving radical, rational, literal, and logarithmic equations
solving systems of linear equations
Students should be able to evaluate arbitrary arithmetic expressions.
Students should be able to use computing machinery to visualize the relationship between variables.
Students will demonstrate an understanding of the concepts of equations, inequalities, graphing, polynomials, proportions and rational expressions.
Students will demonstrate the ability to use algebra to relate mathematics to the real world.
Students will demonstrate the ability to use reasoning skills to analyze information and problem solve.
Students will have developed basic intermediate algebra manipulative skills (working with number sets and interval notation
students will have developed problem-solving skills via exploration of application and modeling problems involving the use of linear, quadratic, polynomial, and rational functions, rational expressions, systems of linear
The learner will solve equations or systems of equations involving rational expressions.
The learner will solve quadratic equations.
The learner will utilize graphing techniques to write, graph, and solve equations.

Translate applied problems in one or two variables and provide a solution through algebraic manipulation
Translate word problems into mathematical equations, apply algebraic techniques to solve the equations, then provide a solution to the original problem.
Understand the concepts of linear functions, quadratic functions and their applications
Understand the concepts of rational and real numbers, and their arithmetic; variables, algebraic expressions and equations; functions and their numerical, algebraic, geometric and verbal representations
Understand the numeric, symbolic, and graphical representations of functions
Use a calculator to perform basic operations and find powers, roots, and logarithmic values
Use properties of logarithms
using functional notation; factoring polynomials
Utilize simple right triangle trigonometry (sine, cosine, tangent) in problem solving
Utilize the properties of special right triangles in problem solving
Utilize the Pythagorean Theorem in problem solving
Write equations of lines and graph them
Write the equation of a line satisfying given conditions involving ordered pairs and/or slope
Write the equation of a line, parabola, or circle given certain conditions

Appendix B

Minnesota State Colleges and Universities Intermediate Algebra Standards Setting Meeting, Maple Grove, MN, October 22-23, 2012

This meeting was the last phase of the test development for the new customized intermediate algebra exam to be piloted Spring 2013. The members of the committee set additional standards to assure the questions developed and the weighting of questions met the desired outcomes of placing students appropriately into our mathematics sequences of courses system wide.

Performance Level Descriptors – Ready for College Level Algebra (JOE)

1. Able to solve multi-step linear equations and inequalities, including some with rational coefficients
2. Able to add, subtract, multiply polynomials and divide polynomial by monomial
3. Factor polynomials by greatest common factor
4. Factor polynomials in quadratic form into the product of binomials including the difference of perfect squares
5. Can identify graphical properties of linear and quadratic functions
6. Translates between graphical, tabular, and symbolic forms of linear functions
7. Solves systems of 2 x 2 linear equations with integer coefficients using any method
8. Use laws or rules of integer exponents to simplify expressions
9. Add, subtract, multiply, and simplify rational expressions
10. Solves simple radical equations with only one square root that simplifies into a linear equation
11. Solves quadratic equations with rational solutions
12. Recognize function notation and able to evaluate a function for a specified value (polynomial, radical, rational, exponential, absolute value)
13. Solve for one variable in a multi-variable equation in terms of other variables
14. Translates phrases to Algebraic expressions and equations
15. Solving equations with absolute value
16. Simplify radical expressions
17. Simplify numerical expression with rational exponents
18. Convert expressions between radical form and exponential form

Appendix C

<table>
<tr><th>Strands</th><th>Objectives</th><th>Examples/Explanations</th></tr>
<tr><td rowspan="3">A. Number and Operations
Students will have a robust understanding of real numbers and demonstrate computational fluency with real and complex numbers. Strong number sense and meaningful understanding of arithmetic operations support the ability to estimate, reason and operate flexibly in quantitative situations and serve as the foundation of algebraic thinking.</td><td>a. Represent, compare and classify real and complex numbers.</td><td>
<table>
<tr><th></th><th>Natural Number</th><th>Whole Number</th><th>Integer</th><th>Rational Number</th><th>Irrational Number</th><th>Complex Number</th></tr>
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<tr><td>b. Perform operations on real numbers including addition, subtraction, multiplication, division, and powers.</td><td></td></tr>
<tr><td>c. Perform operations on complex numbers such as</td><td>Ex. $(3+2i)(2-4i)$
Ex. $(3-2i)-(5-6i)$
Ex. i^{100}</td></tr>
</table>

	addition, subtraction, and multiplication.	
B. Modeling Students will engage in modeling activities that are connected to contexts found in applied situations and settings. Modeling experiences support the development of mathematical reasoning and conceptual understanding and provide the opportunity to formulate, solve and reflect on mathematical problems and processes.	a. Translate phrases to algebraic expressions and sentences to equations or inequalities.	Ex. The difference of a number and two, divided by five Ex. The sum of 3 times a number and 10, subtracted from 9 times a number Ex. Twice a number increased by 4 is at least 15. Ex. Five more than three times a number is the same as three subtracted from the number.
	b. Solve applied mathematical problems.	To pertain to the school's offerings for programs and may include but are not limited to: uniform motion, work/rate, mixture, and geometry problems which result in linear or nonlinear equations and can be solved by the methods discussed in this document.
	c. Interpret solutions in the context of the problem.	Ex. A $6.00 toll is charged to cross the toll road. A six-month reduced-fare pass, costing $50.00, reduces the toll to $2.00 each time. A six-month unlimited-trip pass costs $300 and allows free crossings. How many crossings in six months does it take for the reduced-face pass to be the more economical choice? Solution: More than 12 trips and fewer than 125 trips.

C. Symbolism and Algebraic Expressions Students will represent mathematical situations and structures symbolically, and flexibly operate with algebraic expressions. Connecting concepts and techniques to overarching algebraic structures through abstraction and	a. Add, subtract, and multiply polynomials.	Ex. $(-2x+6y)+(4x-8y)$ Ex. $(x-2y)(3x^2+xy-9y^2)$ Ex. $(3x+7)(5x+5)$ Ex. $(x-y)-(4x-3xy+2y)$ Ex. $(2x - 5)^3$
	b. Divide polynomials by a monomial or binomial	Ex. $\frac{12x^3-28x^2}{4x^2}$ Ex. $\frac{3t^3-11t-12}{t+4}$ Ex. $(2x^2+x-15)\div(x+3)$

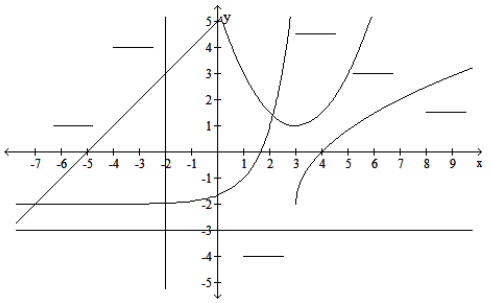
generalization of patterns and relations supports meaningful understanding of mathematics as an integrated whole. Symbolic representation of mathematical concepts and structures and their connections to graphical, numerical and contextual representations serve as an important component to the foundation of algebraic thinking and reasoning.	c. Factor a variety of polynomials i. Whose terms have a common factor ii. With integer coefficients and are quadratic in form using a variety of methods (e.g. greatest common factor, trial and error, difference of squares, grouping, sum and difference of two cubes).	Ex. $2x^2y^2 + 4xy^2 - 30y^2$ Ex. $6x(x + 8) - (x + 8)$ Ex. $10ab - 14a - 25b + 35$ Ex. $x^2 - 9x + 18$ Ex. $10p^2 + 21pq - 10q^2$ Ex. $x^2 - \frac{9}{16}$ Ex. $2x^5 - 16x^2$
	d. Use rules of exponents to simplify expressions including rational exponents.	Ex. $\frac{(0.02\times10^4)}{(4\times10^{-2})}$ Ex. $4^{-2} + 2^{-3}$ Ex. $8x^{-6}$

		Ex. $\left(\frac{1}{4}\right)^{-3}$ Ex. $(-5x^4y^{-3})^2(2x^2y)^{-3}$ Ex. $\left(\frac{a^{-1}b^2}{a^3b^{-2}}\right)^{-3}$ Ex. $\frac{(6x^{-4}y^3)^{-2}}{3x^5y^{-4}}$ Ex. $(2m)^0 + 9^{\frac{1}{2}}$ Ex. $x^{\frac{1}{2}}\left(x^4 + x^{\frac{2}{3}}\right)$
	e. Add, subtract, multiply, divide and simplify rational expressions.	Ex. $\frac{2x}{x^2-4} + \frac{5}{2-x} - \frac{x+3}{2+x}$ Ex. $\frac{2x+4y}{2x^2+5xy+2y^2} \cdot \frac{4x^2-y^2}{8x^2-8} \div \frac{x^2-xy-6y^2}{x^2-6xy+9y^2}$ Ex. $\dfrac{\dfrac{8}{x^2-x-20}}{\dfrac{x+6}{x-5}}$
	f. Simplify algebraic radical expressions and perform operations with radicals.	Ex. $\frac{\sqrt{315x^5}}{\sqrt{5x^{-5}}}$ Ex. $2\sqrt{8} - 4\sqrt{45} - \sqrt{32}$

		Ex. $\sqrt[4]{16x^4}$ Ex. $\sqrt[3]{81b^4}$
	g. Convert expressions between radical form and exponential form	Ex. $25^{\frac{3}{2}}$ Ex. $\sqrt[3]{-64x^5y}$
D. Algebra Equations and Inequalities Students will operate and reason within mathematical situations and contexts that are represented through algebraic equations and inequalities. Meaningful understanding of mathematical relationships that are defined through equality and inequality support the ability to flexibly approach the manipulation of such	a. Solve multi-step linear equations, including some with rational coefficients	Ex. $2(x+1) = -3(9x-2)+5x$ Ex. $\frac{y-3}{3} - \frac{y-2}{2} = -1$ Ex. $\frac{1}{3}(4x+8)-16 = \frac{-4}{9}(9x-12)$
	b. Solve multi-step linear inequalities, including some with rational coefficients.	Ex. $\frac{7}{8}(5x-4)-17 \geq 38$ Ex. $\frac{2}{3}\left(\frac{1}{2}-x\right)-\frac{3}{4}\left(x+\frac{2}{3}\right) \leq 2x-1$ Ex. $10 \leq 2x+1 \leq 15$ Ex. $4x-3 \geq 5$ or $5-2x \geq -5$
	c. Solves systems of 2 x 2 linear equations with rational	Ex.

relationships in the processes of problem solving and modeling.	coefficients using all the following methods i. Graphing ii. Elimination (addition/linear combination) iii. Substitution	$\begin{cases} 4x+5y=-9 \\ 6x-3y=-3 \end{cases}$ Ex. The paths of two ships are tracked on the same coordinate system. One ship is following a path described by the equation $2x + 3y = 6$, and the other is following a path described by the equation $y = \frac{2}{3}x - 3$. a) Graph both equations on the same coordinate system. Is there a possibility of a collision? b) Is a collision a certainty? To include systems with solutions, no solutions, and dependent systems (infinite solutions).
	d. Solve rational equations.	Ex. $\frac{2}{3}-\frac{1}{t}=\frac{7}{3t}$ Ex. $\frac{2}{x-2}+\frac{1}{x+4}=\frac{x}{x^2+2x-8}$ To include identifying restricted values of x.
	e. Solve radical equations.	Ex. $\sqrt{3x-2}+6=11$ Ex. $\sqrt{3x+7}-x=1$ Ex. $\sqrt{2x+3}+\sqrt{x-2}=4$ To include examples that result in extraneous roots.

	f. Solve quadratic equations with real solutions and complex solutions using the following methods: i. Factoring ii. Square Root Property iii. Completing the square iv. Quadratic formula	Ex. $x(x+6) = 7x + 2$ Ex. $(x - 5)^2 = 24$ Ex. $4(3x - 5)^2 = 49$ Ex. $3x^2 + 5x - 28 = 0$ Ex. $x^2 - 4x + 29 = 0$ (complex roots)
	g. Solve literal equations.	Ex. $V = \pi r^2 h$ for r. Ex. $Q = \frac{F-A+K}{A}$ for A. Ex. $r = \sqrt[3]{\frac{3V}{4\pi}}$ for V. Ex. $V_1 = \frac{BI_o R_o}{R + R_o}$ for R_o
	h. Solve absolute value equations.	Ex. $5 - 2\|3x - 4\| = -5$ Ex. $\|2x + 3\| + 6 = 5$

<table>
<tr>
<td rowspan="2">E. Functions and Relations
Students will demonstrate an understanding of the concepts of relations and functions through translations within and among multiple modes of representations including symbolic, graphical, numerical, and contextual. Relations and functions are central structures to algebra and understandings of their properties and generalizations about families of relations and functions support the development of mathematical thinking and problem solving</td>
<td>a. Recognize and graph the following functions
i. linear
ii. quadratic
iii. absolute value
iv. square root
v. exponential</td>
<td>Match each graph with its equation. Write the letter preceding the equation on the space provided.
1) a) $y = mx + b$, b) $x = a$, c) $y = a^x + b$, d) $y = \sqrt{x - a} + b$, e) $y = b$, f) $y = Ax^2 + Bx + C$
 
Sketch the graph of the following functions:
i. linear
ii. quadratic
iii. absolute value
iv. square root
v. exponential</td>
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<td>b. Translate between graphical, tabular, verbal and symbolic forms of functions.</td>
<td>Ex. The value of a vehicle x years after it is purchased is given by the function $f(x) = -2500x + 21{,}000$. Make a table of values and sketch the graph. Interpret the slope (depreciation) and the x and y intercepts of the function.
Ex. In a time long ago (called “the 90s”) there was such a thing as long-distance phone service. A typical monthly long-distance plan charged $4.95 in addition to 8 cents for every minute of calling time used. Let t represents the amount of calling time used, in minutes, and C represents the total monthly long-distance bill. Create a table</td>
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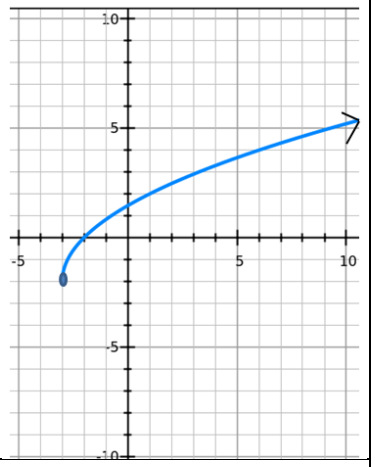
		of values. Write an equation that shows the relationship between time and cost. Sketch the graph.
	c. Recognize functional notation, identify domain and range, and evaluate a function for a specified value.	Ex. Given $f(x) = x^2 + x - 5$, find $f(-3)$, $f(0)$, and $f(3)$. Ex. Given $g(x) = 5\|x - 2\|$, find $g(-3)$, $g(0)$, and $g(3)$. Ex. Given $f(x) = \sqrt{x-4} + 5$ find $f(5)$, $f(4)$, and $f(0)$, and find the domain of $f(x)$. Ex. Given $g(t) = \frac{t^2-2}{t+6}$ find $g(-2)$, $g(0)$,and $g(3)$and find the domain of $g(t)$. Ex. Find the domain and range of the function shown. 
	d. Perform operations with functions.	Ex. $P(x) = 4x + 1; Q(x) = 2x - 5$ Find a. $(P + Q)(x)$ b. $(P - Q)(x)$ c. $(P \cdot Q)(x)$